23.11. The electric field is uniform and goes from higher to lower potential. We also know that $V = -\int_a^b \vec{E}\cdot d\vec{l}$

a) $V_{BA} = 0$ (The electric field is perpendicular to the position vector between A and B)

b) $V_{CB} = V_C - V_B = (-4.20\ N/C)\,(4-(-3)) = -29.4\,V$ (considering the difference in x coordinate of C, B as $\vec{E}$ lies along X axis)

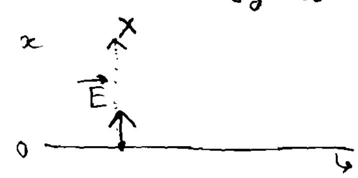
c) $V_{CA} = V_C - V_A = (V_C - V_B) + (V_B - V_A) = -29.4 + 0 = -29.4\,V$

23.12. The electric field produced due to a uniform large plate is $|\vec{E}| = \frac{\sigma}{2\epsilon_0}$

The electric field points away from the surface if the charge on it is positive.

$V(x) - V(0) = \; V(x) - V_0 = -\int_0^x \vec{E}\cdot d\vec{l} = -\int_0^x \frac{\sigma}{2\epsilon_0}\,\hat{i}\cdot dx\,\hat{i} = -\frac{\sigma x}{2\epsilon_0}\Big|_0^x$

$= -\frac{\sigma x}{2\epsilon_0}$



$\therefore V(x) = V_0 - \frac{\sigma x}{2\epsilon_0}$

23.14. a) The potential due to a charged sphere of radius r_0 on its surface having a charge Q is given by - $V_0 = \frac{Q}{4\pi r_0 \epsilon_0}$

$\therefore Q = 4\pi\epsilon_0 r_0 V_0$

charge density $(\sigma) = \frac{\text{charge}}{\text{Surface area of sphere}} = \frac{Q}{4\pi r_0^2} = \frac{4\pi\epsilon_0 r_0 V_0}{4\pi r_0^2} = \frac{V_0\epsilon_0}{r_0}$

$= \frac{(680\,V)\,(8.85\times10^{-12}\,C^2/Nm^2)}{(0.16\,m)} = 3.761\times10^{-8}\,C/m^2$

b) The potential due to charge Q at a distance r from its centre -

$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{4\pi\epsilon_0 r_0 V_0}{4\pi\epsilon_0 r} = \frac{r_0 V_0}{r}$ $\quad\therefore r = \frac{r_0 V_0}{V} = \frac{(0.16\,m)(680\,V)}{(25\,m)}$

$= 4.352\,m$

23.19. a) If we want to find the electric field due to a sphere having charge Q, outside the sphere, we can consider it as a point charge and the electric field would be given as $|\vec{E}| = \frac{Q}{4\pi\epsilon_0 r^2}$, $r > r_0$

$V = -\int_\infty^r \vec{E}\cdot d\vec{r} = -\int_\infty^r \frac{Q}{4\pi\epsilon_0 r^2}\,dr = \frac{Q}{4\pi\epsilon_0 r}\Big|_\infty^r = \frac{Q}{4\pi\epsilon_0 r}$

b) We now need to find electric field inside the sphere and hence the charge enclosed by a ~~s~~ gaussian sphere of radius ~~l~~ r. $(r < r_0)$

Charge enclosed $= \dfrac{Q}{\frac{4}{3}\pi r_0^3} \times \frac{4}{3}\pi r^3 = \dfrac{Q r^3}{r_0^3}$

$$\int \vec{E} \cdot d\vec{S} = \frac{Q r^3}{\epsilon_0 r_0^3}$$

$$|\vec{E}| \times 4\pi r^2 = \frac{Q r^3}{\epsilon_0 r_0^3} \qquad \therefore \ |\vec{E}| = \frac{Q r}{4\pi \epsilon_0 r_0^3} \qquad (r < r_0)$$

$$V(r) = V(r_0) - \int_{r_0}^{r} \vec{E} \cdot d\vec{l} \qquad \left(\because V(r_0) = -\int_{\infty}^{r_0} \vec{E} \cdot d\vec{l}\right)$$

$$= \frac{Q}{4\pi \epsilon_0 r_0} - \int_{r_0}^{r} \frac{Q r}{4\pi \epsilon_0 r_0^3} \, dr$$

$$= \frac{Q}{4\pi \epsilon_0 r_0} - \frac{Q}{4\pi \epsilon_0 r_0^3} \left. \frac{r^2}{2} \right|_{r_0}^{r} = \frac{Q}{4\pi \epsilon_0 r_0} - \frac{Q}{8\pi \epsilon_0}\left(\frac{r^2}{r_0^3} - \frac{r_0^2}{r_0^3}\right)$$

$$= \frac{Q}{4\pi \epsilon_0 r_0}\left(1 + \frac{1}{2}\right) - \frac{Q}{8\pi \epsilon_0} \frac{r^2}{r_0^3}$$

$$= \frac{3 Q r_0^2}{8\pi \epsilon_0 r_0^3} - \frac{Q r^2}{8\pi \epsilon_0 r_0^3} = \frac{Q}{8\pi \epsilon_0 r_0^3}\left(3 - \frac{r^2}{r_0^2}\right)$$

c)

$$V(r) = \frac{Q}{8\pi \epsilon_0 r_0^3}\left(3 - \frac{r^2}{r_0^2}\right), \quad r < r_0$$

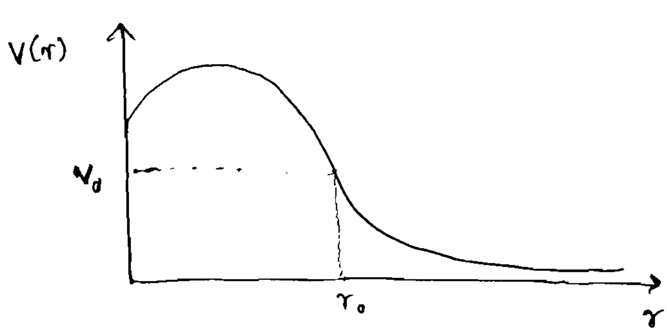
$$= \frac{Q}{4\pi \epsilon_0 r}, \quad r > r_0$$

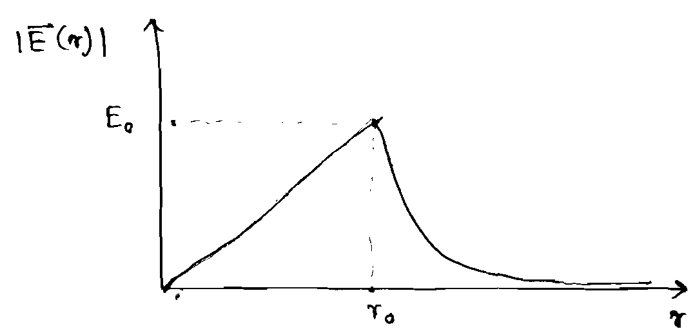
$$= \frac{Q}{4\pi \epsilon_0 r_0}, \quad r = r_0$$

$$|\vec{E}(r)| = \frac{Q r}{4\pi \epsilon_0 r_0^3}, \quad r < r_0$$

$$= \frac{Q}{4\pi \epsilon_0 r^2}, \quad r > r_0$$

$$= \frac{Q}{4\pi \epsilon_0 r_0}, \quad r = r_0$$





23.22. a) To find the electric field we consider the net charge enclosed in each region and then divide by $4\pi r^2$ (surface area of the gaussian sphere) to get the value of electric field.

For $r > r_2$, charge enclosed $= Q + \frac{Q}{2} = \frac{3Q}{2}$.

$$|\vec{E}| = \frac{3Q}{8\pi\epsilon_0 r^2} \quad , \quad r > r_2$$

For $r_1 < r < r_2$, its a conductor and electric field inside it is zero.

For $r < r_1$, charge enclosed is $\frac{Q}{2}$.

$$|\vec{E}| = \frac{Q}{8\pi\epsilon_0 r^2} \quad , \quad r < r_1 .$$

b) For $r > r_2$, we can consider the net charge as a point source and find potential accordingly.

$$V(r) = \frac{3Q}{8\pi\epsilon_0 r} = \frac{\frac{3Q}{2}}{4\pi\epsilon_0 r} \quad , \quad r > r_2 .$$

c) Inside a conductor $(r_2 > r > r_1)$ the potential remains constant along with zero electric field. Thus the potential on the surface is the one inside the conductor as well.

$$V(r) = \frac{3Q}{8\pi\epsilon_0 r_2} \quad , \quad r_1 < r < r_2 .$$

d) For this region, we use -

$$V(r) = -\int_\infty^r \vec{E}\cdot d\vec{r} = -\left[\int_\infty^{r_2} \vec{E}\cdot dr + \int_{r_2}^{r_1} \vec{E}\cdot d\vec{r} + \int_{r_1}^{r} \vec{E}\cdot d\vec{r}\right]$$

$$= -\left[\int_\infty^{r_2} \frac{3Q}{8\pi\epsilon_0 r^2}\, dr + \int_{r_2}^{r_1} 0 \cdot d\vec{r} + \int_{r_1}^{r} \frac{Q}{8\pi\epsilon_0 r^2}\, dr\right]$$

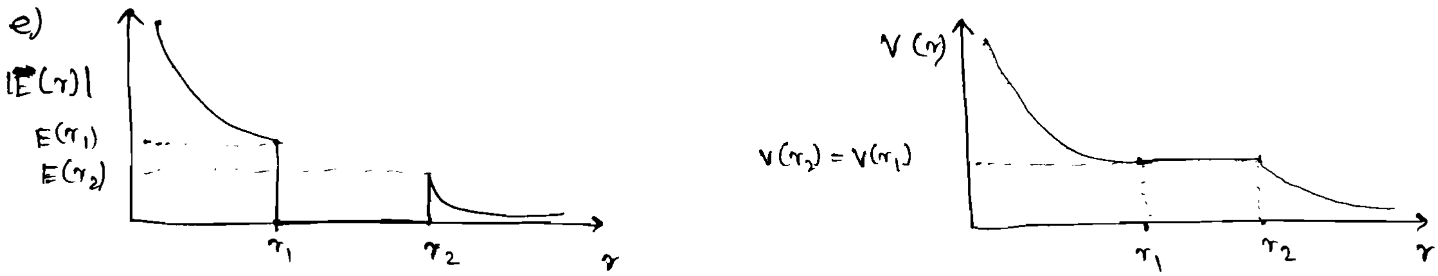
$$= -\left[-\frac{3Q}{8\pi\epsilon_0 r}\Big|_\infty^{r_2} + 0 - \frac{Q}{8\pi\epsilon_0 r}\Big|_{r_1}^{r}\right]$$

$$= \frac{3Q}{8\pi\epsilon_0 r_2} + \frac{Q}{8\pi\epsilon_0 r} - \frac{Q}{8\pi\epsilon_0 r_1}$$

$$= \frac{Q}{8\pi\epsilon_0}\left(\frac{3}{r_2} - \frac{1}{r_1} + \frac{1}{r}\right) \qquad \because r_2 = 2r_1$$

$$= \frac{Q}{8\pi\epsilon_0}\left(\frac{3}{2r_1} - \frac{1}{r_1} + \frac{1}{r}\right) = \frac{Q}{8\pi\epsilon_0}\left(\frac{1}{2r_1} + \frac{1}{r}\right) \quad , \quad r < r_1$$

e)



23.25. a) The potential due to a point charge at a distance r away is -

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

$$\therefore Q = 4\pi\epsilon_0 r V = \frac{1}{8.99\times10^9\ N\,m^2/C^2} \times$$

$$= (8.99\times10^9\ N\,m^2/C^2)\times\frac{1.6\times10^{-19}\ C}{0.50\times10^{-10}\ m} = 28.77\ V$$

b) The potential energy of the electron is the charge of the electron times the potential due to the proton.

$$U = QV = -(1.6\times10^{-19}\ C)\ (28.77\ V) = -4.6\times10^{-18}\ J$$

it is an electron now

23.30. There is no loss of energy in the system. The initial potential energy of the system will be converted to their kinetic energy. Moreover the particles are at rest initially that is they have zero momentum. Hence when they move apart they will have velocity in opposite direction of same magnitude.

Initial electrostatic energy $= Q\left(\frac{Q}{4\pi\epsilon_0 r}\right) = \frac{Q^2}{4\pi\epsilon_0 r}$

Kinetic energy of two particles $= 2\times\frac{1}{2}mv^2 = mv^2$

$$mv^2 = \frac{Q^2}{4\pi\epsilon_0 r}$$

$$\therefore v = \sqrt{\frac{Q^2}{4\pi\epsilon_0 m r}} = \sqrt{\frac{(5.5\times10^{-6})^2\ (8.99\times10^9\ Nm^2/C^2)}{(10^{-6}\ Kg)\ (0.065\ m)}} = 2\times10^3\ m/s$$

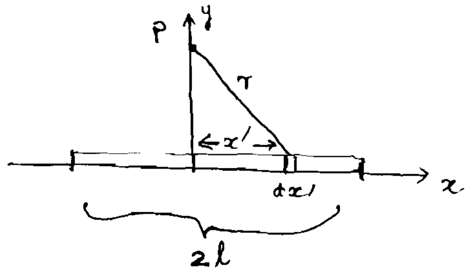
23.36. In this example the center where we are finding the potential is at equal distances (radius) from all points on the semicircle.

For a semicircle of radius r_0 its circumference is $\pi r_0 = l$.

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r} = \frac{1}{4\pi\epsilon_0 r_0}\int dq = \frac{Q}{4\pi\epsilon_0 r_0} = \frac{Q}{4\epsilon_0 l}$$

($\because r = r_0$ for all)

23.38.



we choose a small length element dx' at a distance x' from center of rod and find the potential due to that. Total charge on rod $= Q$.

Charge on length $dx' = \frac{Q}{2l} dx'$

Distance between the charge and the pt. P is $r = \sqrt{x'^2+y^2}$.

$$\therefore V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\int_{-l}^{l} \frac{Q}{2l}\frac{dx'}{\sqrt{x'^2+y'^2}}$$

$$= \frac{Q}{8\pi\epsilon l}\left[\ln\left(\sqrt{x'^2+y'^2}+x'\right)\right]_{-l}^{l} = \frac{Q}{8\pi\epsilon_0 l}\ln\left(\frac{\sqrt{l^2+y^2}+l}{\sqrt{l^2+y^2}-l}\right)$$

23.55. The gain in the kinetic energy comes from the loss in potential energy as there is no energy loss associated with the system. The helium atom has a charge $2e$.

$$\Delta K.E. = -\Delta U = -qV$$

$$\therefore V = -\frac{\Delta K.E.}{q} = -\frac{125\times10^3 eV}{2e} = -62.5\times10^3 V$$

23.58. The charge of the particle does not affect its kinetic energy and not needed here.

$K.E. = \frac{1}{2}mv^2$ (v being velocity)

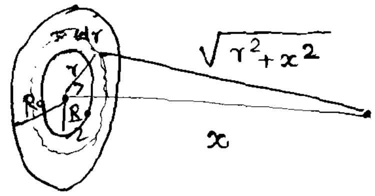
$$\therefore v = \sqrt{\frac{2K.E.}{m}} = \sqrt{\frac{2(5.53\times10^6 eV)(1.6\times10^{-19} J/eV)}{6.64\times10^{-27} Kg}} = 1.63\times10^7 m/sec$$

23.75. The electrons starts off with a velocity v and come to rest due to the electric interaction. The loss in kinetic energy is the one which is the work done by the electrostatic field.

$$\frac{1}{2}mv^2 - 0 = qV$$

$$\therefore v = \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2\times(-1.6\times10^{-19} c)(-3.02V)}{9.11\times10^{-31} Kg}} = 1.03\times10^6 m/s$$

23.82.



charge density $= \sigma = \dfrac{Q}{\pi R_0^2 - \dfrac{\pi R_0^2}{4}}$

$= \dfrac{4Q}{3\pi R_0^2}$

We choose a small ring of thickness dr at a radius r and find potential due to that ring and finally add it up for all such values of r.

Amount of charge on it $= \underbrace{(2\pi r)}_{\text{circumference of ring}} \underbrace{(dr)}_{\text{Thickness of ring}} \sigma$

$= 2\pi\sigma\, r\, dr$

$$dV = \frac{1}{4\pi\epsilon_0}\frac{2\pi\sigma r\, dr}{\sqrt{r^2+x^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{r\,dr}{\sqrt{r^2+x^2}}$$

$$V = \int_{R_0/2}^{R_0} \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{r\,dr}{\sqrt{r^2+x^2}}$$

$r^2 + x^2 = t^2$

$r\,dr = t\,dt$

r	R_0	$R_0/2$
t	$\sqrt{R_0^2+x^2}$	$\sqrt{x^2 + R_0^2/4}$

$$= \int_{\sqrt{\frac{R_0^2}{4}+x^2}}^{\sqrt{R_0^2+x^2}} \frac{2\pi\sigma}{4\pi\epsilon_0}\cdot\frac{\cancel{t}\,dt}{\cancel{t}}$$

$$= \frac{2\pi\sigma}{4\pi\epsilon_0}\, t\,\Big|_{\sqrt{\frac{R_0^2}{4}+x^2}}^{\sqrt{R_0^2+x^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0}\left(\sqrt{R_0^2+x^2} - \sqrt{\frac{R_0^2}{4}+x^2}\right)$$

$$= \frac{2\cancel{\pi}}{4\cancel{\pi}\epsilon_0}\cdot\frac{\cancel{4}Q}{3\cancel{\pi}R_0^2}\left(\sqrt{R_0^2+x^2} - \sqrt{\frac{R_0^2}{4}+x^2}\right)$$

$$= \frac{2Q}{3\pi\epsilon_0 R_0^2}\left(\sqrt{R_0^2+x^2} - \sqrt{\frac{R_0^2}{4}+x^2}\right)$$

24.2. We know the relation between charge and capacitance -

$Q = CV = (12.6\times10^{-6}\,F)(12) = 1.51\times10^{-4}\,C$

24.5. The total charge in the whole process is conserved. In the second configuration the voltage across the two capacitors should be same as they are connected by wires without any resistance between them.

In the initial case -

$Q = C_1 V_{1\,initial}$.

In the final case -

$V_{final} = \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}$ $\quad\therefore\ Q_1 + Q_2 = (C_1 + C_2)V_{final}$

$Q = Q_1 + Q_2$

$C_1 V_{initial} = (C_1 + C_2) V_{final}$

$\therefore C_1 (V_{initial} - V_{final}) = C_2 V_{final}$

$$\therefore C_2 = C_1 \left(\frac{V_{ini.}}{V_{fin.}} - 1 \right) = (7.7 \times 10^{-6} F) \left(\frac{125V}{15V} - 1 \right) = 5.6 \times 10^{-5} F$$
$$= 56 \mu F$$

24.15. We assume an uniform electric field between the plates i.e. $V = Ed$.

Using $C = \frac{Q}{V} = \frac{A \epsilon_0}{d}$

$$\therefore Q_{max} = CV = \frac{A \epsilon_0}{d} \times V = \frac{A \epsilon_0}{d} \times E d = A \epsilon_0 E$$
$$= (8.85 \times 10^{-12} F/m)(6.8 \times 10^{-4} m^2)(3 \times 10^6 V/m) = 1.8 \times 10^{-8} C$$

24.22. a) Net capacitance when capacitors are connected in parallel -

$C_{eq} = C_1 + C_2 + C_3 + C_4 + C_5 + C_6 = 6(3.8 \times 10^{-4} F) = 2.28 \times 10^{-5} F$

b) Net capacitance when capacitors are connected in series -

$$\frac{1}{C_{net}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5} + \frac{1}{C_6} \right) = \frac{6}{3.8 \times 10^{-6} F}$$

$$\therefore C_{net} = \frac{3.8 \times 10^{-6}}{6} F = 6.3 \times 10^{-7} F$$

24.29. a) C_1, C_2 are in series which is in parallel with C_3. This is then in series with C_4. We now find equivalent capacitance.

$$\frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C} \qquad \therefore C_{12} = \frac{C}{2}.$$

$$C_{123} = C_{12} + C_3 = \frac{C}{2} + C = \frac{3C}{2}$$

$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4} = \frac{2}{3C} + \frac{1}{C} = \frac{5}{3C}$$

$$\therefore C_{1234} = \frac{3C}{5}$$

b) The charge on capacitor C_{1234} is $Q = C_{1234} V = \frac{3CV}{5}$. The same charge is placed on the other series components of C_{1234}.

$$Q_{123} = \frac{3}{5} CV = C_{123} V_{123} \qquad \therefore V_{123} = \frac{2}{5} V$$

$$Q_4 = \frac{3}{5} CV = C_4 V_4 \qquad \therefore V_4 = \frac{3}{5} V$$

The voltage across the equivalent capacitance C_{123} is the voltage across both of its parallel components as they are in parallel.

However the sum of the charges across two parallel components of C_{123} is the same as the total charge on the two components $\frac{3}{5}CV$.

$V_{123} = \frac{2}{5}V = V_{12}$ $\quad Q_{12} = C_{12}V_{12} = \frac{C}{2} \cdot \frac{2}{5}V = \frac{CV}{5}$

$V_{123} = \frac{2}{5}V = V_3$ $\quad Q_3 = C_3 V_3 = C\left(\frac{2}{5}V\right) = \frac{2CV}{5}$.

Charge on C_{12} is the charge on both its series components -

$Q_{12} = \frac{1}{5}CV = Q_1 = C_1V_1 \quad \therefore V_1 = \frac{V}{5}$

$Q_{12} = \frac{1}{5}CV = Q_2 = C_2V_2 \quad \therefore V_2 = \frac{V}{5}$.

24.31. When the switch is down the charge on C_2 is given as -

$Q_2 = C_2 V_0$

When the switch is moved up, the charge on C_2 will flow to C_1 until the voltage on two capacitors is equal.

$V = \frac{Q_2'}{C_2} = \frac{Q_1'}{C_1} \quad \therefore Q_2' = Q_1' \frac{C_2}{C_1}$

Since charge is conserved, the total charge should equal the initial charge.

$Q_1' + Q_2' = Q_2$

$Q_1' + Q_1'\frac{C_2}{C_1} = C_2 V_0$

$\therefore Q_1'\left(1 + \frac{C_2}{C_1}\right) = C_2 V_0$

$Q_1' = \frac{C_2 V_0}{(C_1 + C_2)} \times C_1$ $\quad Q_2' = \frac{C_2}{C_1} \times \frac{C_1 C_2 V_0}{(C_1 + C_2)}$

$= \frac{C_2^2 V_0}{(C_1 + C_2)}$

24.41. The energy stored in a capacitor is given as -

$E = \frac{1}{2}CV^2 = \frac{1}{2}(2.8 \times 10^{-9}F)(2200V)^2 = 6.8 \times 10^{-3} J$

24.44. a) The charges on the capacitors remain constant when the distance is changed between the capacitors. However the capacitance changes.

$$\frac{U_2}{U_1} = \frac{\frac{1}{2}C_1V_1^2}{\frac{1}{2}C_2V_2^2} = \frac{\frac{1}{2}\frac{Q^2}{C_2}}{\frac{1}{2}\frac{Q^2}{C_1}} = \frac{C_1}{C_2} = \frac{\frac{A\epsilon_0}{d}}{\frac{A\epsilon_0}{3d}} = \frac{3}{1}$$

$(\because V = \frac{Q}{C})$

b) Amount of work done is the change in the energy in capacitors -

$$U_2 - U_1 = 3U_1 - U_1 = 2U_1 = 2\frac{Q^2}{2C_1} = \frac{Q^2}{C_1} = \frac{Q^2}{\frac{A\epsilon_0}{d}}$$

$$= \frac{dQ^2}{A\epsilon_0}$$